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THE CORE OF PROGRESS

With increased industrial activity has come increased sharpness of competition. Increase of competition has made necessary more and more of standardizing processes whereby are furnished more precise measures of service and product of service. Increase of standardization has inevitably brought with it greatly increased use of mathematics. Never, as now, has the science of statistics been so in demand,—and statistics is but a form of mathematics. No producer is now satisfied to ask: Is this method more efficient than that? He wants a numerical measure of the efficiency of a method, a mathematical translation of "more." To compare, to weigh, to value, to count, to arrange, to place in order, to put in one—one correspondence—these are the inner essence of every program of science, of every efficiency-seeking step of industry. Thus is mathematics the final motor of the world's advance. —*S. T. S.*

A USE OF HABIT

A recent writer in Science directs attention to a certain type of psychology which has been successfully applied in the field of music. Certain students of piano were required throughout their training periods to strike only the correct notes, as deter-

mined by the music page. At first this must often have meant a long pause between finger actions on the keyboard. But, in the end were built up habits of striking, not wrong, but right keys, so that even under the stress of a public performance, few, if any, errors of technique were made. The writer—a mathematician—inquires: In the same manner, why may not beginners in mathematics, or number work, be habituated to placing on paper, or on the blackboard, only the correct digit, the correct sum, product, quotient, etc.? The question is worthy of the most serious study. We are tempted to add the following: An effect of such a habit, once it has been firmly fixed, would be a greatly enlarged appreciation by the student of the IMPORTANCE of ERROR-FREE WORK, as far as it may be possible to have it. —*S. T. S.*

SELF-HELP

All adequate teaching methods must be inspired by one fundamental objective, namely, to stimulate the student to a control of his own mental development. But self-help is, of necessity, rooted in INTEREST. Thus is the creation of interest in the student mind for the subject matter, where this interest does not exist, the supreme task of an instructor. In individual cases this interest already exists. Such may, legitimately and safely, be let alone. They offer no teaching problem. By far the large majority of students, up to the sophomore year, particularly in the field of mathematics, particularly in these times when the call of athletics and the social program to the average youth is strong, are deficient in this interest. Hence effective teaching of mathematics was never so difficult as now. We may even say: never, as now, so dangerously difficult. To the thoughtful mathematics teacher the meaning of this phrase is manifest and needs no explanation. If personality can be made a teaching factor, then let it be unlimbered and made to count. If the language gift is there, let it be used to invest the lesson material with every possible grace and charm, making, for that purpose, a fine art of the lecture. But let us study all devices. In every class are some whose permanent interest has been started the moment they are acclaimed before their fellows for having successfully done a diffcult piece of work, for example for having

solved a hard problem. The wise teacher will endeavor to discover all such and work his psychology to the limit.—*S. T. S.*

THE DEFINITION

Frequently is it found more difficult to make a satisfactory definition of the familiar thing than of the unfamiliar. Such terms as length, surface, number, area, solid, line, are in common use by educated and uneducated classes. Yet even the trained mathematician may experience trouble when he undertakes to formulate definitions of some of these terms in a manner not open to criticism. Indeed important branches of mathematics are based, to a considerable extent, on analytic re-castings of the meanings of concepts which, for the average lay mind, already carry clear and definite meanings. One such branch is known as The Foundations of Geometry. If there is one word which is more commonly used than any other it is very probably the word "number." But let the unwary and inexpert layman undertake to formulate explicitly a consistent meaning of the term and he will at once begin to sense unexpected troubles. Similar complications will be found to arise in connection with a host of other very much used concepts. For example, what is the true length of a rubber band, or even of an ordinary tape line? Truly was the mathematician correct who said: That branch of mathematics is an important one in which we lay down DEFINITIONS of things! —*S. T. S.*

ANNOUNCEMENTS

The next annual meeting of the National Council of Teachers of Mathematics will be held February 22 and 23, in Hotel Statler, Cleveland, Ohio. The general theme of the meeting will be announced later. The theme of the last meeting (at Boston) was "Mathematics in Modern Life."

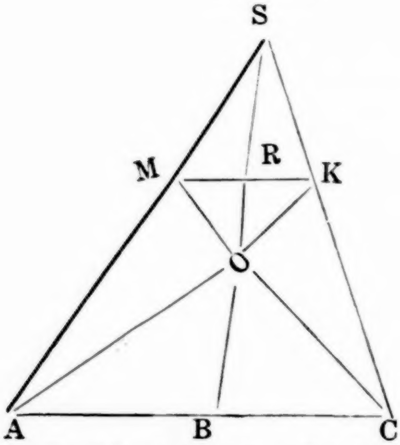
It is advised that those desiring hotel reservations should make them early. Official blanks for doing this will be furnished by President H. C. Barber, Exeter, New Hampshire, to all who apply for them.

A recent note from President Maizlish requests us to announce again that the Louisiana Academy of Sciences will hold

its next annual meeting at Lafayette, La., in conjunction with the Louisiana-Mississippi M. A. of A. meeting at the same place.

A PROBLEM IN PROJECTIVE GEOMETRY*

By MARGUERITE ZEIGEL
Mississippi Delta State Teachers College



Problem: To bisect the parallel bases of a trapezoid by means of a straight edge.

Let ACMK be any trapezoid with the parallel bases AC and MK. Draw AK and MC and call the intersection O. Extend AM and CK until they intersect at S. Connect S and O. This line meets MK in R and AC in B. Then BR is the required line since R and B are the midpoints of the parallel bases MK and AC respectively.

Proof: Let us consider the quadrangle MOKS. Since two of the opposite sides meet AC in A, two sides in C, one in B, and one in an ideal point D, the points A, B, C, D by definition form a harmonic range.

Therefore, B is the midpoint of AC by the following theorem. If in a harmonic range ABCD the point B lies midway between the two conjugates A. and C, then the fourth point D lies at an infinite distance and conversely, if one of the points D lies at infinity, its conjugate B is the point midway between the others, A and C.*

From the two following pairs of similar triangles:

$\triangle AOC \sim \triangle MOK$; $\triangle MOR \sim \triangle OBC$, we have $\frac{AC}{BC} = \frac{MK.}{MR}$

Since B is the midpoint of AC, then from the above proportion R is the midpoint of MK.

*Elements of Projective Geometry—Cremona. Third Edition p. 48—No. 59.

FAILURES IN TRIGONOMETRY

By J. P. COLE
Louisiana Polytechnic Institute

I believe it is high time that we school people take into consideration something that is worthy of our most serious attention. First, I shall state facts, and then give conclusions. My experience covers a period of nine years (1919-1928). In that time I have taught 22 sections of trigonometry with a total of 446 students. Of this number, 181 failed, approximately 40%. The least number of failures in any class was 7 out of 23 (about 30%), and the largest number of failures was 12 out of 19 (about 63%). During this same period I have had 22 sections of college algebra with 481 students. Out of this number 98 failed (about 20%). The least number of failures in any class was 2 out of 17, and the largest number of failures was 7 out of 21. During the same period I have had sections in solid geometry, analytics, calculus, differential equations, and college geometry, but the percentage of failures did not approach that in trigonometry. My conclusion is that something is out of adjustment somewhere. There are three things to cause this trouble: the student, the teacher, or the book. In observing the student I find that, with very few exceptions, the least number of failures in trigonometry is greater than the largest in any other subject. One might say the cause is the teacher. This was given serious consideration, and in order to convince the most skeptical, data were gathered from my fellow teachers here. Their conclusions were the same as mine. Without exception their largest failures were always in trigonometry. I could give figures, but the results would be as above. Therefore it muse be the book. Observe any trigonometry that you may have near at hand. Look through it hurriedly. Does it

not look disconnected? Take, for instance, the chapter on the functions of any angle. In looking over my shelves of books I pick up the first trigonometry I find. It is an old one just revised. There are 20 pages devoted to this chapter. I believe it could be condensed into four and perhaps less. Take the chapter on logarithms. There are 24 pages devoted to it. In my opinion it should not be in trigonometry at all. So far as I know, all institutions teach algebra before trigonometry, and we find this subject occupying a whole chapter in algebra. But suppose it must be kept in trigonometry if for no other reason than its close relation to the subject. Five pages could cover it with all excess written matter omitted. There are 12 pages on natural functions. Two pages could cover this and leave space for some historical remarks. There are 25 pages devoted to the trigonometric functions of acute angles. Certainly that is too much. Each chapter could be condensed 50% or more and then there would be a meaning for all that is written. My review of the whole situation is: We need some one to give us a good condensed book in this subject. Let it be mathematical, with not too much reading matter. As a usual thing our first impressions are the most lasting. Hence the student should be presented with a book that looks good, to say the least. I frankly believe I could get better results if I omitted the text altogether, and used a study-recitation period instead of the present recitation hour. It is hoped that the above brief frank discussion of this problem will be the means of a real solution of it. Comments are invited.

PROBLEM SOLVING

By W. PAUL WEBBER
Louisiana State University

The statement that James is as tall as Henry is an equation. For, it states that the height of James equals the height of Henry. Again, to say that an orange costs six cents is to state the equation,

Cost of an orange equals six cents.

Pupils may learn to find x in

$$3x-4=2x+7$$

without ever suspecting that the above statements are in fact equations. Whenever, by words or symbols, two things or qualities are to be equal, or equivalent, the statement is an equation. It might be an interesting exercise to search a page of a newspaper for such statements. It is possible that equations are commonly used where we least suspect them.

It is the special purpose of elementary algebra to deal with equations and their use in solving problems. A large part of problem solving consists in discovering an unknown quantity, or several unknowns, in a statement of relations and implications popularly known as applied problems. There are certain laws according to which equations may be manipulated so as to obtain the value of the unkown. It is assumed here that a sufficient knowledge is already in the pupil's possession.

The solution of a problem may be resolved into the following steps:

1. Determination of all the equations in the statement of the problem, either by direct declaration or by implication.

2. Symbolization of the unkown quantity or quantities.

3. Writing the equations in symbolic (algebraic) form.

4. Seeing that the number of distinct equations equals the number of unknowns to be determined.

5. Sometimes it may be necessary to employ relations (equations) not given in the problem in order to obtain the necessary equations. This we shall call extra-problem knowledge. Often this is the chief difficulty in solving a problem.

6. Application of the technique of formal algebra to solve the equations.

Examples

(1) A merchant finds seventy bills in his cash register, which amount to $150. He also observes that in it are only five-dollar and one-dollar bills. Determine how many of eacl kind there are.

1. The equations in the problem are:

The value of a certain number of five-dollar bills plus the value of a certain number of one-dollar bills is $150. The number of five-dollar bills added to the number of one-dollar bills is 70.

2. Symbolizing the unknowns,

Let x be the number of five-dollar bills,
Let y be the number of one-dollar bills.

3. Writing the equations in algebraic form,
   (1) $x+y=70$
   (2) $5x+1y=150$ or $5x+y=150$

4. The number of equations is equal to the number of unknowns.

5. Solving the equations it is found that $x=20$, $y=50$.

The result may be checked by substitution in the original equations.

(2) The area of a square is 729 square units. The diagonal is 8.284 units longer that a side. Find the side and diagonal of the square.

1. The length of a side plus 8.284 equals length of diagonal.
   The area equals 729.
2. Let x represent the length of a side.
   Let y represent the length of the diagonal.
3. (1) $y=x+8.284$
   (3) Area=729
4. The number of symbolic equations is not sufficient.
5. The extra-problem knowledge needed is:
   Area of square equals the square of a side.
   Square of the diagonal equals twice square of a side or twice the area.

Now write
   (3) $y^2=2x^2$ or $y^2=2 \cdot 729=1458$

6. Solving the last equation with the equation $y=x+8.284$ gives $x=20$, $y=28.284$

The result may be checked by substitution in equations (1) and (2).

There are two ways in use for obviating the difficulty of extra-problem knowledge in the statement of the problem: One is to give all such knowledge in the statement of the problem. The other is to give a tabulation of all such knowledge for the entire text in an appendix. The amount of such extra problem knowledge that can be carried in the memory is small. The first expedient often makes the statement of a problem tediously long or renders the use of certain problems inadvisable. The second expedient has great possibilities.

SOME CLASS ROOM JOTTINGS

By HARRY GWINNER
University of Maryland

My inability to handle classes as I would like to do causes me to worry. But I am improving, and notwithstanding the criticism directed against me that I am too particular, I feel that I may, in the years to come, handle classes so efficiently that not a minute of the precious credit hour shall be wasted.

The classes under my immediate jurisdiction average about twenty students, and I have board space for all, limiting the width to 24 inches per student. This arrangement has a tendency to prevent spreading and carelessness due to the penalty of erasing and doing over.

The room assigned to my sections has two divisions of 15 seats each and I arrange students in these according to ability, one on the east side of the room and the other on the west side, and the divisions are known as the East and West. As the students are balanced in the seating, checking of the absent ones is easy. This division is followed, approximately, when students are sent to the board. In general, problems are assigned so that adjacent students will not have the same problems, and the east group will not have the same problems as the west group.

Each student is provided with a rule 24 inches long, hung at a convenient place so as not to interfere with board space, and chalk is placed in the rail below the board. The rules are yard sticks supplied through the courtesy of hardware and dry goods friends. They are shortened so as not to stick in the other fellow's eye.

In oral tests, the student orating is sometimes interrupted by coaching from the side lines, but zeros judiciously applied to both generally prevent the spread of the disease.

This "thing" you call textbook at times gives me mental indigestion. I have one text in mind.—I wrote to the author for the solution of a problem, and his reply is as follows: "The example in question crept into the book by mistake. As far as I know, it cannot be solved by elementary methods. It is

my intention to replace it by another in the next reprint, if the book is put through." From the same author in the same text is the following "—is another example which cannot be solved by elementary methods and will be replaced in the next edition of the text."

Two other problems in the same text were so unsatisfactory that they failed to be reappointed. Another text used by freshmen contained an astronomical problem about which two instructors remarked that they did not know of what the authors were talking.

Thank you, kind reader, for your sympathy—I need it.

MATHEMATICS—THE MOST ATTRACTIVE SUBJECT OF THE CURRICULUM

J. E. DOTTERER
Professor of Mathematics, Manchester College, Ind.

Is the above statement true? By most people it probably would be considered false. Mathematics, as all other subjects, has its advocates and its enemies. The inference of many popular educators has been that high school pupils take mathematics because it is a required subject. The same idea has been carried over into college mathematics. It is considered by some to be one of the least popular branches of learning. Others have a marked indifference towards it. Of course many like it. If it is not liked by the masses of students, what is the reason? (Personally I do not believe there is such a distaste for the subject as is sometimes believed). Is it due to inability on the part of a large number of students to master it? Is it due to the fact that the content is naturally dry and uninteresting to them? Is it due to the supposition that the subject matter has little application in the affairs of everyday life? Is it due to the fact that we as teachers of the subject fail to bring to our students the best there is in this great field of knowledge?

The last four questions may all be discussed at length. They should be clearly understood by all teachers. Some students give as their reason for not liking the subject that their fathers and grandfathers did not make good in it. No doubt this frequently produces a natural prejudice in a student. Others give

the reason that they see no practical value in it. And yet, when they are pinned for a definite explanation, they most frequently acknowledge that there is about as large a percentage of usable knowledge in mathematics as in any other field of the whole curriculum. The natural dryness of the subject is very closely related to the other two just mentioned. Each year I meet quite a number of college freshmen who do not like mathematics. It is part of my program in each interview to endeavor to discover the reason for this. The answers are many and are largely covered by the questions poposed above. But, if the questions are driven home with force, the answer usually simmers down to this: I had a very poor teacher in algebra or geometry or both who gave me a natural distaste for the subject. Sometimes it goes back as far as eighth grade arithmetic. However, the poor teaching is not all in the high schools. Much of it is in college.

There are two kinds of mathematics teaching. The effect of the one is to produce interest, enthusiasm and mental power on the part of the student. The other most frequently drives students from the class except those who like mathematics in spite of the teacher. The teacher is a salesman. According to the laws of salesmanship he should be full of his subject, enthusiastic about it, able to show its usefulness and power to his class. Mathematics has a message for practically all classes of students—those who are scientifically inclined and those who are leaning toward vocations where mathematics is only indirectly used. They all need it. It is our task as teachers to get it across—whether we are teaching in high school or in college.

Now to the subject heading. If mathematics is taught only as juggling with a mass of symbols, it is not likely to be interesting to the large group of students. If it is taught as a great field of knowledge, as an instrument solving the problems of nature and life, interest and enthusiasm will be kindled. Those prejudiced against it by earlier influences will be won over. It has been my privilege to receive testimonies from students who hated mathematics upon coming to college but whose vision regarding the subject was changed. They left college with a high appreciation for this science.

The problem after all is largely a problem of teaching. The personality of the teacher is frequently the factor which deter-

mines whether the students like mathematics or not. So, also, are the methods used and the subject matter presented. No business man can afford to send out one dissatisfied customer. Neither can we as mathematics teachers afford to send out many who do not like our subject. Our material is worth getting out to the masses. It can be made the most attractive course in the curriculum in both high school and college. Let us make it a living subject and not a mere juggling with dead symbols. I want to challenge the readers of this paper to write for publication many articles on the various means of making mathematics interesting. Interest can be produced without destroying the rigor of the subject.

A PROBLEM IN ANALYTIC GEOMETRY

By H. L. SMITH
Louisiana State University

It is customary in textbooks on analytic geometry to handle angles by means of their slopes. It has always seemed to the writer that for this purpose the cosine has advantages over the tangent, and it is the object of this note to exhibit these advantages by the treatment of a definite problem. We shall need a preliminary formula.

The cosine of an angle. Let it be required to find the cosine of a geometric angle $P_1P_0P_2$ in terms of the coordinates $(x_1\ y_1)$, $(x_0,\ y_0)$, $(x_2,\ y_2)$ of the points P_1, P_0, P_2, respectively. Set

$$(1)\qquad \begin{cases} h_1=x_1-x_0, & k_1=y_1-y_0 \\ h_2=x_2-x_0, & k_2=y_2-y_0 \end{cases}$$

so that

$$(2)\qquad h_1-h_2=x_1-x_2,\ k_1-k_2=y_1-y_2$$

Moreover let d_0, d_1 d_2 be the sides of triangle P_0 P_1 P_2 which are opposite respectively to vertices P_0, P_1, P_2. Then by (1)

$$(3)\qquad d_1=\sqrt{h_1{}^2+k_1{}^2},\ d_2=\sqrt{h_2{}^2+k_2{}^2}$$

and by (2)

$$(4)\qquad d_0{}^2=\sqrt{(h_1-h_2)^2+k_1-k_2)^2}$$

Then in triangle P_0 P_1 P_2, by the law of cosines from trigonometry,

$$(5)\qquad \cos P_1\ P_0\ P_2=(d_1{}^2+d_2{}^2-d_0{}^2)/(2d_1\ d_2)$$

But, in view of (3), (4), the equation (5) simplifies to

(6) $\cos P_1 P_0 P_2 = (h_1 h_2 + k_1 k_2)/(d_1 d_2)$.

This is the required formula.

Formulation and solution of the problem. Let it be required to find the locus of a point P such that the angle A P B is constant, where A and B are two fixed points. Let the coordinates of A, B, P, be $(-c, o)$, (c, o), (x, y), respectively, $(c>o)$ and let a denote the angle A P B. Then the equation of the locus of P is

$$\cos a = [(x+c)(x-c)+y^2]/[\sqrt{(x+c)^2+y^2}\sqrt{(x-c)^2+y^2}]$$

or

(7) $(\cos a)\sqrt{(x^2+y^2+c^2)^2-4c^2x^2} = x^2+y^2-c^2$,

subject to the restriction that (xy) shall not be either $(-c, o)$ or (c, o). It remains to simplify (7).

Now the locus of (7) satisfying the above restriction consists of those points of the locus of

(8) $(\cos^2 a)[(x^2+y^2+c^2)^2-4c^2x^2] = (x^2+y^2-c^2)^2$

which are such that $(x^2+y^2-c^2)$ has the same sign as $\cos a$. But (8) may be written in the form

$$(x^2+y^2-c^2)^2-4(\operatorname{ctn}^2 a)c^2y^2=0$$

or

(9) $[x^2+y^2-c^2+2(\operatorname{ctn} a)c|y|]\,[x^2+y^2-c^2-2(\operatorname{ctn} a)c|y|]=0$,

and the points on the locus of (9) for which $x^2+y^2-c^2$ has the same sign as $\cos a$ are (since $\operatorname{ctn} a$ has the same sign as $\cos a$) precisely all the points of the locus of

$$x^2+y^2-c^2-2(\operatorname{ctn} a)c|y|=0$$

or its equivalent

(10) $x^2+(|y|-c \operatorname{ctn} a)^2=(c \csc a)^2$

except those (A and B) for which y is zero. Hence (10), subject to the restriction that y shall not be zero, is the equation of the locus of P.

Interpretation of equation (10). In the half-plane in which $y>o$ equation (10) becomes

(11) $x^2+(y-c \operatorname{ctn} a)^2=(c \csc a)^2$,

which is the equation of a circle with radius $c \csc a$ and centre $(o, c \operatorname{ctn} a)$. In the half-plane in which $y<o$ (10) becomes

(12) $x^2+(y+c \operatorname{ctn} a)^2=(c \csc a)^2$,

which is the equation of a circle with radius ($c \csc a$ and centre $(o, -c \operatorname{ctn} a)$. Hence the locus of (10) consists of that part of the locus of (11) which lies in the half-plane $y>o$ together with

that part of the locus of (12) which lies in the half-plane $y<o$.

Remarks on the solution. The reader should note that there are no exceptions to the above solution, as is the case in the usual solution. Moreover the work was of such a character that the extraneous locus introduced by squaring (7) was finally eliminated. Finally, attention is called to the method for determining the locus of an equation in which the running coordinates are affected by absolute value signs. The reader may find it interesting to determine the loci of other equations of this type, such as, for example.

$$|x|+|y|=1,$$
$$(|x|-h)^2+(|y|-k)^2=r^2,$$

and so on.

TYPES OF THINKING IN ALGEBRA

By CORA DROZ BROUSSARD
George Washington High School, Los Angeles, Cal.

This paper is an investigation of the thinking processes involved in the learning of algebra for the purpose of decreasing the difficulties that high school pupils encounter in the study of this subject. Most teachers have difficulty in presenting the subject- matter in such a way that all can readily grasp the operations involved. If they knew more about what goes on in the child's mind while working, better provision could be made for him in the presentation of the subject.

Some of the questions that this research endeavors to answer are: How do pupils think while learning algebra? Do they think in the most economical and successful way? Do they follow the teacher's line of reasoning or do they have a method of their own? Does the beginner think from concrete to abstract or the reverse? Is it desirable to bridge the gaps between algebra and geometry and algebra and arithmetic? Does algebra have a logic of its own or are there familiar methods of thinking that need only to be pointed out and adopted to algebraic tasks? Does ability to learn algebra vary much? Can these differences be utilized in finding the most economical method by learning what methods of thinking algebra are characteristic of slow and what of fast learners?

Not very much experimentation and investigation have been done along this line until recently, that is, within the last decade. Mr. Thorndike has written a very comprehensive article on the Nature of Algebraic Abilities. He says that, in the past, two forces determined the actual content with which abilities were trained. These consisted of custom which resulted in the attempt to make algebra parallel arithmetic and the continuance of puzzle problems. The new algebra "omits any unnecessary problems." He divides the process of problem solving into three groups—that in which the student organizes data to express a desired result; a second in which he is taught to solve already organized formulae or equations; or a third in which he is required to solve equations of the form $y=ax+b$, or $y=x^2+ax+b$.

The same author has written an article on the Psychology of Errors in Algebra. In this paper he enumerates the types of errors possible—accidental and recurring. "The outcome of the psychological analysis of errors shows that there is a need for a psychological analysis of algebraic processes into the constituent connections or bonds involved." He also discovered there was need of drill or practice in various processes in order to make them more fixed. Class procedure should also be reorganized so that a "spirited atmosphere" will be created, that is, timing all functional activities and shifting rapidly in the type of mental processes will add to the interest in the subject and thus guarantee better progress therein.

Mr. Thorndike has also written a discussion on the "Psychology of Problem Solving" in which he states that "problem solving should be taught so as to get a general relation between variables so that any problem may be solved. At present we are too concrete." He also states that interest may be increased if a few problems of puzzle and mystery are introduced or if the teacher will, at times, allow students to elect some themselves.

Another of Mr. Thorndike's discussions is that on the "Psychology of the Equation." "There are two uses of the equation," he states. One is the "organization of data in such a way as to indicate the operations required to find a certain result," and the other is "an expression of a relation between one or more variables." He says that the algebra of a generation ago was free from confusion because it did not attempt to

teach the equation as the expression of a general relation between a variable and one or more other variables. No graphs and Cartesian coordinates were introduced to make it further confusing.

While observing algebra classes at work, Mr. Thorndike discovered that a majority of pupils lacked a mastery of the elements of the subject. As a result of his many years of experience he has been able to give us a few important principles to guide us in the strengthening of the fundamental bonds. The student, he says, should acquire the habit of expecting the operations to give a trustworthy, useful result. They should be given the impression that the letters used are real numbers. The process of learning should be infused with interest, he states, so that pupils **care** about obtaining right answers. Another of his suggestions is to provide aids to bridge the transition from learning A and B and C and D to learning to operate A and B together and C and D together, and later A, B, C, and D all together.

Another investigator in this field is Mr. Charles H. Butler. His series of articles on the Role of Memory in Algebra are very inspiring. He believes that memory is one of the greatest factors in algebraic thought. These discussions reveal the fact (through author's experimentation) that certain types of memory are found in this subject, while certain other types find relatively little application. For example "sheer recall" is found in all the types of thought while "parrot-like" verbal recall is found only in the drill aspect of algebraic thought. (I regret that these discussions have not been completed at the time of the writing of this paper.)

While teaching algebra I observed that children have difficulty in manipulating exponents, substitution, quadratics and thought problems. A test in each of these types showed that such was the case in all of my classes. Thinking that the cause of the difficulty lay in my method of presentation I planned a test to be given to the first and second year algebra pupils in the parishes of East Baton Rouge and Ascension, Louisiana.

Great care was taken to select typical problems in the four fundamental operations, substitution, simple equations, factoring, clearing fractions, fractional equations, radicals, simultaneous linear and quadratic equations, exponents, expansion, formu-

lae, parentheses and reasoning problems. The test consisted of forty-three problems, the above types being represented by from one to three. Mimeographed copies were sent to the schools in said parishes, each child was given one. Blanks requiring the name, age, school, parish and the length of time the subject has been studied, were provided for on the back of each test. Directions for conducting the test were stated on a separate sheet as follows:

To the Teacher in Charge:-

Please distribute papers face downward.

Instruct the pupils as follows:-

1. Fill in the blanks with the required information.
2. Read over **carefully all** of the problems in the test and check those that you think you can work.
3. After doing this, go back and work the ones you have checked.
4. Then, spend the rest of the time trying to work the ones not checked.
5. If a problem is exceedingly hard, make a question mark by it.

Repeat these instructions before they begin. Please give **absolutely** no help.

Time limit **sixty** minutes.

The following is a copy of the test given:-

1. $4x+20x+7x+11x=$
2. If $a=2$ and $b=3$, what does $8a+2b$ equal?
3. Express a number 7 less than m; y less than m.
4. Sove for t: $3t+6=27$.
5. $7(x+3)=$
6. $(m+n)^2=$
7. $a^3 \times a^7=$
8. Factor: $5a^2+25a^3$.
9. Clear fractions but do not solve: $\dfrac{1}{x}=\dfrac{7}{8}$.
10. Solve for x: $\dfrac{x+3}{15}+\dfrac{x+2}{4}=6$.
11. Solve for t:

$d=ir.$

12. Solve for m:

$m^2=m+2.$

13. Solve the following system of equations:

$x+y=7$

$2x-3y=4$

14. Simplify: $\sqrt{32}$.
15. $5a+7a-4a=$
16. If $m=3$ and $n=2$, what does m^2n+mn^2 equal?
17. From $3a^2+4b-3c$ take $5a^2-2b+4c$.
18. Solve for x: $3x+2=2x+5$.
19. $-m(m-n)+5=$
20. $(y+7)(y-6)=$
21. $\frac{z^6}{z^4}=$

2. Factor: x^2-49.

23. Clear fractions but do not solve: $\frac{2x-1}{2}+\frac{x-2}{3}=\frac{2}{3}$
24. Divide 84 into two parts such that the fraction formed by these parts is $\frac{5}{7}$.
25. $C=\frac{Ka-b}{a}$. Solve for K.
26. A schoolroom contains 48 seats, and the number in a row is 4 less than twice the number of rows. How many seats in a row?
27. Solve the following system of equations:

$p+2q=17$

$3p-q=2$

28. Simplify:

$\sqrt{m^5n}$

29. $3m+4n-7m+6m-5n=$
30. The sum of two numbers is 110. If the smaller is s, what is the larger?
31. Divide a pole 20 ft. long into two parts so that one part shall be 4 times as long as the other.

32. The sum of two numbers is 14 and the larger exceeds the smaller by 2. Find the numbers.
33. $(3a+b)\ (3a-b)=$
34. $(y^4)^3=$
35. Solve for r:

$$i=\frac{prt}{100}$$

36. What number is it whose square plus the number itself equals 42?
37. Solve the following system of equations:

$$3r-3s-4=0$$
$$4r+5s=30.$$

38. Simplify:

$$\sqrt{a/b}$$

39. Sove by completing the square:

$$x^2+3x-28=0.$$

40. Simplify:

$$\sqrt[3]{5x^2/16y^2}$$

41. Factor: x^4+x^2+1.
42. Simplify: $\sqrt{80}-\sqrt{180}$
43. Solve graphically:

$$x+y=4$$
$$x^2+y^2=16$$

The results of this test prove very interesting. They are divided into three groups—percentage of errors in relation to months of training, in relation to chronological age, and also in relation to index of brightness. The percentages of the boys and girls were found separately, but owing to the fact that very little difference exists, they were later combined into these three tables:

Table I: Per cent of errors in relation to months of training.

Table II: Per cent of errors in relation to chronological age.

Table III: Per cent of errors in relation to index of brightness.

(Discussions of the Tables will be made in the November issue of the Mathematics News Letter.)

PROBLEMS

Proposed by H. L. Smith: Show that the inequality

$$|(x_1-x_2)\ (y_1+y_2)+(x_2-x_3)\ (y_2+y_3)+(x_3-x_1)\ (y_3+y_1)|$$
$$\leq|x_1-x_2|\ |y_1-y_2|+|x_2-x_3|\ |y_2-y_3|+|x_3-x_1|\ |y_3-y_1|$$

holds for all real values of the six letters involved, and give a geometric interpretation.

Proposed by G. M. White. Given: A circle whose diameter is d, and whose center is o.

Another circle drawn, inside the first, whose diameter is d′ and whose center is o′,

A line is drawn from any outside point C, cutting the two circles at a and m respectively.

A line mb is drawn making angle Cmo′=angle o′mb.

Coo′ is a straight line.

Required: The distance oo′ so that the sum of the lines am and mb will be a constant.

Note: This is a problem of light, mn is a convex mirror and the line Cmb is a beam of light originating at C.

If this problem has no solution as stated, can it be solved with the line Co given?

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